

SKY COVERAGE AND BURST REPETITION

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ABSTRACT

To investigate the repeater content of gamma ray burst samples I develop two models where sources burst at a constant average rate. I find that the sky coverage affects the number of repeaters in a sample predominantly through the detector livetime, and that the number of bursts in the sample is the primary parameter. Thus the repeater content of burst samples should be compared within the context of a repetition model; a direct comparison between two samples is possible only if the samples have similar sizes. The observed repeater fraction may not be the actual fraction if the sources burst on average less than once during the detector livetime. Sources which burst repeatedly during active phases separated by more than the observation period must be treated separately.

Subject headings: gamma rays: bursts

1. INTRODUCTION

Whether the observations from the Burst and Transient Source Experiment (BATSE) on the *Compton Gamma Ray Observatory* (CGRO) show that classical gamma ray bursts repeat is of great current interest because most of the models for bursts at cosmological distances destroy the burst source in producing the burst. The reports of repeaters in BATSE's 1B (i.e., first) catalogue of 260 bursts (Fishman et al. 1994) were based on spatial (Quashnock & Lamb 1993) and spatial-temporal (Wang & Lingenfelter 1993, 1995) clustering. The spatial clustering is not evident in the 2B (i.e., second) catalogue (which includes the bursts of the 1B catalogue—Meegan et al. 1994) of a total of 585 bursts (Meegan et al. 1995a), although whether a weak spatial-temporal clustering signal is present is disputed (Brainerd et al. 1995 do not find the signal that Wang & Lingenfelter

1995 and Petrosian & Efron 1995 find). All repeater signals are absent using the revised burst positions in the 3B catalogue (Meegan et al. 1995b), even for the bursts which were included in the 1B catalogue. While the BATSE observations do not show convincingly that burst sources repeat, they do not rule out such repetitions. Because of the large uncertainties in the positions of bursts localized by BATSE (less than 2° systematic and from less than 1° to more than 15° statistical, depending on the burst intensity—Meegan et al. 1995b), studies of the repeater content of the BATSE catalogs generally do not identify individual repeating sources but instead determine whether the spatial clustering is greater than expected for an isotropic distribution of events, or alternatively what fraction of the observed events could be from repeaters. These statistical methods can only address what is actually in the data. Explicit or implicit in deriving the physical implications of these analyses are assumed models of source repetition. Searches for spatial-temporal clustering assume that burst sources repeat during active phases while the searches for spatial clustering are sensitive to any repetition pattern. Here I investigate the models behind previous studies, and develop two simple models of burst repetition. Note that I explore the connection between the observed and actual source repetitions, and not the accuracy in determining a burst sample’s observed repeater content.

The reduced sky coverage during the second year of BATSE’s operation has been invoked to explain the weak or absent repeater signal during BATSE’s second year under the assumption that burst sources repeat. The bursts in the 2B catalogue but not in the 1B (usually referred to as 2B-1B) were accumulated during this second year. I define sky coverage f_s as the fraction of the sky BATSE views on average; the sky coverage is less than 1 as a result of Earth blockage, SAA passages, telemetry gaps, etc. Indeed, during BATSE’s first year $f_s \sim 1/3$ and during the second $f_s \sim 1/4$. Some of the repeater studies model the repetitions among the observed bursts without regard for the actual repeater pattern (Strohmayer, Fenimore & Miralles 1994; Meegan et al. 1995a; Brainerd et al. 1995), while others model the repetitions in the underlying burst population, whether observed or not. Here I argue that deriving a physically meaningful statement regarding the presence of repetitions depends on a model of such repetitions. In particular, the comparison of different burst ensembles (e.g., the 1B vs. 2B-1B catalogues) requires an assumed repetition model.

To make this analysis more concrete, I first develop in detail two simple models: a “stochastic” model where the probability of a source bursting per unit time is constant (§2) and a model where the source bursts at a constant rate (§3). Based on these models, I comment on features of more general repeater models (§4). Finally I discuss the use of repetition models in studies of the repeater content of the BATSE bursts (§5). Great care must be taken in defining quantities so that it is clear whether a quantity includes all bursts

during an observation period or only those which are observed, and whether a repeating source is one which eventually will repeat or one actually observed to repeat.

2. STOCHASTIC BURSTS

Assume that each source bursts stochastically at a fixed rate (this is a modeling assumption): the probability of an observable burst within any given time interval is constant and independent of the intensity of, or time since, the previous burst. Since detector thresholds differ, the rate is detector-dependent. Similarly, subsets of a burst sample with different thresholds have different effective rates. A source that bursts only once (i.e., does not repeat) can be treated as having a rate of once per Hubble time. Thus the number of bursts observed from the i th source which bursts at a rate r_i over a time ΔT by a detector with a sky coverage f_s will have a Poisson distribution with mean (and variance) $n_i = r_i f_s \Delta T$. In this model it does not matter whether the detector's livetime is contiguous ($f_s = 1$) or highly chopped up ($f_s \ll 1$); only the total livetime $\tau = f_s \Delta T$ is relevant.

For a fixed number of sources N_S , the number of bursts observed during a given livetime τ is

$$\langle N_B \rangle = \sum_{i=1}^{N_S} \sum_{n=1}^{\infty} n \frac{(r_i \tau)^n}{n!} \exp[-r_i \tau] = \sum_{i=1}^{N_S} r_i \tau \quad (1)$$

while the number of bursts from sources which are observed to repeat (i.e., from which two or more events are observed) is

$$\langle N_{B,r} \rangle = \sum_{i=1}^{N_S} \sum_{n=2}^{\infty} n \frac{(r_i \tau)^n}{n!} \exp[-r_i \tau] = \sum_{i=1}^{N_S} r_i \tau (1 - \exp[-r_i \tau]) \quad (2)$$

and the fraction of all events which are observed to originate from repeating sources is

$$f_r = \frac{\langle N_{B,r} \rangle}{\langle N_B \rangle} = \frac{\sum_{i=1}^{N_S} r_i \tau (1 - \exp[-r_i \tau])}{\sum_{i=1}^{N_S} r_i \tau} \quad (3)$$

Thus N_B is proportional (on average) to τ . For those sources with $1/r_i \leq \tau$ few repeaters will be observed ($n_i \leq 1$ and the Poisson probability of 2 or more events will be small). Both the average number of repetitions within a burst sample and the size of the burst sample are functions of τ , and thus a sample's observed repeater content is a function of its size. Note that as τ and the sample size increase, more sources will be observed to repeat and the burst sample will include more repetitions. Of course the ability to detect repetitions in the burst sample may decrease as the sample size increases. For example,

the angular correlation function is inversely proportional to the number of bursts N_B : $w(\theta) \propto [(N_B - 1)\sigma_*^2]^{-1}$, where σ_* is the uncertainty in the burst positions (Meegan et al. 1995a).

Note that the product $r_i\tau$ recurs in all expressions for the number of observed repetitions, bursts observed, etc. In reality, the sky coverage f_s and thus the livetime τ are not constant over the sky. The local value of τ_i (i.e., the value of τ at the location of the i th source) is the relevant parameter, but the above argument can be extended from a distribution of r_i to a distribution of $r_i\tau_i$. Here I will neglect the variations of the sky coverage over the sky. Hakkila et al. (1995) use a sky coverage which varies with declination to constrain the repeater fraction from the observed isotropy assuming a stochastic model.

As an example, assume that there are N_S repeating sources with the same burst rate r as well as a class of nonrepeating sources. The nonrepeating sources can be treated as contributing bursts at a constant rate R . Then

$$\langle N_B \rangle = (R + N_S r) \tau \quad , \quad (4)$$

$$\langle N_{B,r} \rangle = N_S r \tau (1 - \exp[-r\tau]) \quad , \quad (5)$$

$$\text{and} \quad f_r = F_r (1 - \exp[-r\tau]) \quad \text{where} \quad F_r = \frac{1}{1 + (R/N_S r)} \quad (6)$$

is the asymptotic value of f_r for large τ when all the repeating sources will have been observed to burst at least twice. This asymptotic F_r is the physically meaningful repeater fraction which analysis of the burst samples should strive to determine. The number of sources observed to repeat is

$$\begin{aligned} \langle N_{S,obs} \rangle &= N_S (1 - (1 + r\tau) \exp[-r\tau]) \\ &= N_S (r\tau)^2 / 2 \quad \text{for} \quad \tau \rightarrow 0 \\ &= N_S \quad \text{for} \quad \tau \rightarrow \infty \quad . \end{aligned} \quad (7)$$

At large livetimes τ every repeating source is indeed observed to repeat.

The average number of events observed from sources observed to repeat (i.e., excluding bursts from repeating sources which are observed to burst only once) is

$$\begin{aligned} \langle n_{obs} \rangle &= \frac{\sum_{n=2}^{\infty} n \frac{(r\tau)^n}{n!} \exp[-r\tau]}{\sum_{n=2}^{\infty} \frac{(r\tau)^n}{n!} \exp[-r\tau]} = \frac{r\tau(1 - \exp[-r\tau])}{(1 - (1 + r\tau) \exp[-r\tau])} \\ &= 2 \quad \text{for} \quad \tau \rightarrow 0 \\ &= r\tau \quad \text{for} \quad \tau \rightarrow \infty \quad . \end{aligned} \quad (8)$$

For small livetimes only two events will be seen from the few sources observed to burst more than once, but eventually all repeating sources will be observed to burst many times.

Strohmayer et al. (1994) used a different repeater fraction which excludes the first burst of each series of two or more bursts from a repeating source. The number of observed bursts which are repetitions of a previously observed burst is $\langle N_{B,r} \rangle - \langle N_{S,obs} \rangle$. The resulting fraction is

$$f'_r = \frac{\langle N_{B,r} \rangle - \langle N_{S,obs} \rangle}{\langle N_B \rangle} = F_r (1 - (1 - \exp[-r\tau]) / r\tau) \quad . \quad (9)$$

Figure 1 shows these various quantities as a function of the expected number of observed bursts per source ($r\tau$) for a model where all sources repeat at a constant average rate r (i.e., there are no nonrepeating sources and $F_r = 1$). The total number of bursts in the ensemble is linearly proportional to $r\tau$.

Note that in this model the repeater content of a burst sample—the number of sources observed to repeat—is a function the sky coverage f_s only through the dependence of the livetime τ on f_s . The number of bursts observed is linearly proportional to the livetime and can be used as a surrogate for the livetime. I therefore expect the repeater character of two samples with comparable numbers for bursts to be the same. Conversely, I expect different observed repeater characteristics for samples with different sizes.

3. BURSTS AT A CONSTANT RATE

Here I assume that the i th source bursts a fixed number of times n_i during a given observation period ΔT . The sky coverage f_s is the probability that any one of these bursts will be observed. The number of bursts observed from any source is characterized by the binomial distribution: the probability of observing $n \leq n_i$ bursts is $P(n) = \frac{n_i!}{n!(n_i-n)!} f_s^n (1 - f_s)^{n_i-n}$. Thus the quantities defined in §2 are

$$\langle N_B \rangle = \sum_{i=1}^{N_S} \sum_{n=1}^{n_i} n \frac{n_i!}{n!(n_i-n)!} f_s^n (1 - f_s)^{n_i-n} = \sum_{i=1}^{N_S} n_i f_s \quad (10)$$

$$\langle N_{B,r} \rangle = \sum_{i=1}^{N_S} \sum_{n=2}^{n_i} n \frac{n_i!}{n!(n_i-n)!} f_s^n (1 - f_s)^{n_i-n} = \sum_{i=1}^{N_S} n_i f_s (1 - (1 - f_s)^{n_i-1}) \quad (11)$$

$$f_r = \frac{\sum_{i=1}^{N_S} n_i f_s (1 - (1 - f_s)^{n_i-1})}{\sum_{i=1}^{N_S} n_i f_s} \quad (12)$$

$$\begin{aligned} \langle N_{S,obs} \rangle &= \sum_{i=1}^{N_S} \sum_{n=2}^{n_i} \frac{n_i!}{n!(n_i-n)!} f_s^n (1 - f_s)^{n_i-n} \\ &= \sum_{i=1}^{N_S} \left(1 - (1 - f_s)^{n_i} - n_i f_s (1 - f_s)^{n_i-1} \right) \end{aligned} \quad (13)$$

$$\langle n_{obs} \rangle = \frac{\sum_{i=1}^{N_S} \sum_{n=2}^{n_i} n \frac{n_i!}{n!(n_i-n)!} f_s^n (1-f_s)^{n_i-n}}{\sum_{i=1}^{N_S} \sum_{n=2}^{n_i} \frac{n_i!}{n!(n_i-n)!} f_s^n (1-f_s)^{n_i-n}} = \frac{\langle N_{B,r} \rangle}{\langle N_{S,obs} \rangle} . \quad (14)$$

As in §2, $\langle N_B \rangle$ is the number of observed bursts, while $\langle N_{B,r} \rangle$ is the number of observed bursts which originate on sources with more than one observed burst. These $\langle N_{B,r} \rangle$ bursts constitute a fraction f_r of the observed bursts $\langle N_B \rangle$. The fraction of all bursts, observed or not, produced by repeating sources is F_r ; asymptotically f_r tends to F_r . The number of sources observed to repeat is $\langle N_{S,obs} \rangle$ and these sources are observed to burst $\langle n_{obs} \rangle$ times.

If there are nonrepeating sources which provide bursts at a rate R and N_S repeating sources which each burst $n_i = n_B$ times during the observation period (the model used by Quashnock [1995a,b] and Graziani [1995]), then

$$\langle N_B \rangle = f_s(R\Delta T + N_S n_B) , \quad (15)$$

$$\langle N_{B,r} \rangle = N_S f_s n_B (1 - (1 - f_s)^{n_B-1}) , \quad (16)$$

$$f_r = F_r \left(1 - (1 - f_s)^{n_B-1} \right) \quad \text{where} \quad F_r = \frac{1}{1 + (R\Delta T / N_S n_B)} , \quad (17)$$

$$\langle N_{S,obs} \rangle = N_S \left(1 - (1 - f_s)^{n_B} - f_s n_B (1 - f_s)^{n_B-1} \right) , \quad (18)$$

$$\langle n_{obs} \rangle = \frac{f_s n_B (1 - (1 - f_s)^{n_B-1})}{1 - (1 - f_s)^{n_B} - f_s n_B (1 - f_s)^{n_B-1}} . \quad (19)$$

Note that the effective rate is $r = n_B / \Delta T$, and the number of bursts observed from repeating sources is $f_s n_B N_S = f_s N_S r \Delta T = N_S r \tau$ where $\tau = f_s \Delta T = f_s n_B / r$ is the livetime. Although there is formally a dependence on f_s beyond its effect on the livetime, the quantities f_r , $\langle N_{S,obs} \rangle$ and $\langle n_{obs} \rangle$ are very nearly the same functions of $f_s n_B$ (which is proportional to the livetime). Figure 2 shows these quantities as a function of $f_s n_B$ for $f_s = 1/4$ and $f_s = 1/3$. The x-axes of Figures 1 and 2 are the same (the number of bursts observed from each repeating source), and the curves for these three models are almost identical above one burst observed per repeating source.

4. MORE GENERAL MODELS

The large range of possible repetition models makes a systematic analysis impossible, but general comments can be made about classes of models. Barring correlations between the sky coverage and the repetition pattern, the repeater content of a burst ensemble is primarily a function of the size of the ensemble for models where the source bursts at an average rate. We showed in detail for two models with an average burst rate that the dependence on the sky coverage is felt through its effect on the livetime. Complications

arise when the repetition pattern has a time scale commensurate with a characteristic time of the sky coverage. For example, if a repetition tends to occur ~ 45 min (i.e., half an orbit) after an initial burst, the Earth may occult one of the bursts for a detector in low Earth orbit.

One might have expected a stronger dependence on the sky coverage f_s which is the probability of observing a given burst. Although the probability of seeing both members of a pair of bursts from a repeating source is f_s^2 , the quantity of interest is the conditional probability of observing repetitions from the source of a previously observed burst. Assuming a repeating source bursts twice during the observation period ΔT , the conditional probability of detecting a repetition of an observed burst is f_s . The sample size accumulated is proportional to $\tau = f_s \Delta T$. Decreasing f_s does reduce the probability of observing a repeater, but it also decreases the number of bursts in the sample. To maintain the sample size, ΔT must be increased as f_s is decreased. But a key assumption was that there were only two bursts during ΔT ; if ΔT increases then the effective burst rate $r = 2/\Delta T$ decreases.

Therefore for f_s to be the probability of observing the repetition of a given burst in a burst sample of a specified size, the effective burst rate must be proportional to the inverse of ΔT . Clearly this is not the case for any repetition model where the rate is constant. However, if the source goes through active bursting phases shorter than ΔT , and the average separation between the active phases is longer than ΔT , then $r \propto \Delta T^{-1}$. This is the class of models which Wang & Lingenfelter (1993, 1995) consider. For example, if a source bursts twice within five days, and then does not burst again for a decade, then $r = 2/\Delta T$ for $\Delta T \sim 1$ yr. However, as ΔT increases we expect an increase in the number of repeating sources which become active during the observation period.

Similarly studies of the sky coverage-dependence of observing repetitions from sources with a fixed number of bursts within the observation period ΔT implicitly assume the burst rate is inversely proportional to ΔT . This assumption excludes fixed rate models, but permits active phase models.

5. DISCUSSION

Above I considered repetitions in the underlying source population, and calculated their observed consequences. This approach provides a basis for comparing different burst samples for consistency in the apparent repeater content, and thus is necessary for resolving whether burst sources repeat. In addition, this formulation permits observational results

(e.g., upper limits on the observed repeater fraction f_r) to be translated into physical constraints on bursts sources (e.g., limits on the rate at which sources burst and the asymptotic repeater fraction F_r). As stated in the Introduction, because of the large uncertain in burst positions, most studies do not attribute individual bursts to specific repeating sources, but instead place statistical constraints on the repeater fraction. I found that the observed repeater content for models with a constant average burst rate is predominantly a function of the size of the burst sample; the dependence on the sky coverage is almost exclusively through the livetime. Thus care must be taken in comparing different size samples.

Some studies start by modeling the repetitions of burst sources, regardless of whether the bursts are observed. In their likelihood analyses of models with repetition Quashnock (1995a,b) and Graziani & Lamb (1995) assumed that each repeating source bursts the same number of times during the observation period (the model in §3). To demonstrate the effect of sky coverage, Meegan et al. (1995a) used this model with each source bursting 10 times during the observation period; for $f = 1/3$ and $f = 1.4$ of the 1B and 2B-1B catalogs this corresponds to $r\tau = 2.5$ and 3.33, respectively, when by Figure 1 f_r will be near its asymptotic value. Hakkila et al. (1995) used the stochastic burst model of §2 and a sky coverage which varies with declination to understand the effects of location errors and sky coverage on the repeater signal.

Other studies model the observed repetitions without regard for the intrinsic behavior of the source population. Some assume that repeaters constitute a fixed fraction of a burst sample. Strohmayer et al. (1994) generated model catalogues by giving each burst a location, with a fraction f'_r being assigned the location of a previous burst. Thus a burst early in the catalogue is more likely to have a repetition than a burst later in the catalogue; it is not clear whether the resulting observed pattern is physically realizable. Meegan et al. (1995a) constrained the possible repeater signal in various subsets of the 2B catalogue with a model where repeating sources each are observed to burst ν times (described as an average) and provided a fraction f of the observed bursts. From all but one subset Meegan et al. found that $f(\nu - 1)^{1.2}/\nu \leq 0.2$. Note that the size of these subsets varies from 185 to 585, and by the analysis above the observed repeater fraction f is not expected to be constant if repeaters are present; the sample size must be considered in evaluating the intrinsic repeater fraction (eq. 6 or 17) implied by these constraints. In addition, some of these subsets use bursts with positional uncertainties less than 9° , as suggested by Quashnock & Lamb (1993). Since the statistical uncertainty in position decreases with increasing burst intensity, the samples with uncertainties less than 9° effectively have a higher burst threshold. Less intense repetitions were not included in these subsets, which consequently have a different repetition rate r for any repeating sources. Brainerd

et al. (1995) considered a repetition model similar to Meegan et al. in an analysis of spatial-temporal clustering, from which they subsequently derived constraints on the true number of repetitions in the 2B catalog (regardless of whether the bursts are observed).

The simple models developed in §2 and §3 have two parameters—the burst rate r and the nonrepeater to repeater fraction $R/N_S r$ —and thus an observable such as f_r from a burst sample does not fully determine the model parameters. For example, in Figure 3 I show the possible values of $R/N_S r$ and $r\tau$ for observed values of f_r using the stochastic model of §2. As can be seen, many repetitions (large $r\tau$) from a small number of repeating sources or few repetitions from many repeaters can result in the same observed repeater fraction f_r . With more observables the parameters can be determined; for the stochastic model, $\langle n_{obs} \rangle$ determines $r\tau$, although it may be difficult to determine $r\tau \leq 1$ since $\langle n_{obs} \rangle$ is very nearly constant at a value of 2 for $r\tau$ in this range. Similarly, the dependencies of the observables on $r\tau$ (or its equivalent) are very nearly the same for the two models developed here, and probably for most constant rate models; it may be very difficult to identify the repetition pattern from observations, particularly if the sky coverage is low. Of course, the repetition pattern can be determined if specific bursts can be attributed to the same source.

To prove the existence of repeaters the observed repeater fraction f_r must be shown to be nonzero. On the other hand, to constrain the allowed repeater population, limits on the actual repeater fraction F_r must be derived from the data.

6. SUMMARY

I have considered the observational consequences of different patterns of burst repetition. I conclude:

1. There is a fundamental difference in the sensitivity to the sky coverage between sources which repeat in active phases separated by time scales greater than the observation period and sources which burst at a constant average rate.
2. For models with sources which burst at a constant average rate the repeater content of a burst sample depends predominantly on the number of bursts in the sample; the sky coverage affects the observations primarily through the livetime, unless the time scales of burst repetition and the sky coverage are correlated.
3. Consequently, similarly-sized burst samples should be compared or a repetition model should be used. Such a model is necessary to constrain the source population based on the observations.

4. Two models are developed in detail. In the first each source bursts stochastically, while in the second each source bursts at a fixed rate.

5. A given observed repeater fraction less than one can result from a small number of repeating sources which burst frequently, or a larger number which burst less frequently.

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Fig. 1.— Repetition quantities as a function of the mean number of observed bursts per source ($r\tau$) for the stochastic model. All sources have the same probability of bursting per unit time, and no nonrepeating sources are included (the fraction of all bursts which originate on repeaters, whether or not the bursts are observed, is $F_r = 1$). The number of observed bursts from sources observed to repeat (i.e., from which there are two or more observed bursts) is n_{obs} (long dashes), while $\langle N_{S,obs} \rangle / N_S$ (dots and dashes) is the fraction of the sources from which repetitions are observed. The fraction of the observed bursts from sources with two or more observed bursts is f_r (solid), while f'_r (short dashes) is the fraction of the bursts which are repetitions of an earlier burst. Note that the first of a series of two or more observed events is included in f_r but not in f'_r .

Fig. 2.— Repetition quantities as a function of the number of observed bursts per source ($f_s n_B$) for constant rate models. In any observation period all sources are assumed to burst the same number of times n_B . Points (the number of bursts per source is an integer) for $f_s = 1/4$ (asterisks) and $f_s = 1/3$ (pluses) are shown. Labels indicate the points for three different quantities: n_{obs} —the number of observed bursts from sources observed to repeat; $\langle N_{S,obs} \rangle / N_S$ —the fraction of the sources from which repetitions are observed; and f_r —the fraction of the observed bursts from sources with two or more observed bursts.

Fig. 3.— Nonrepeater to repeater fraction as a function of the observed number of bursts per source for different values of the repeater fraction f_r in the stochastic model (§2). Each curve is labeled by the value of f_r .